

## Universal Gravitation

### READ

The law of universal gravitation allows you to calculate the gravitational force between two objects from their masses and the distance between them. The law includes a value called the gravitational constant, or "G." This value is the same everywhere in the universe. Calculating the force between small objects like grapefruits or huge objects like planets, moons, and stars is possible using this law.

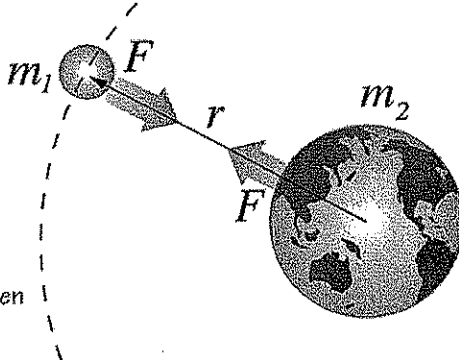
### What is the law of universal gravitation?

The force between two masses  $m_1$  and  $m_2$  that are separated by a distance  $r$  is given by:

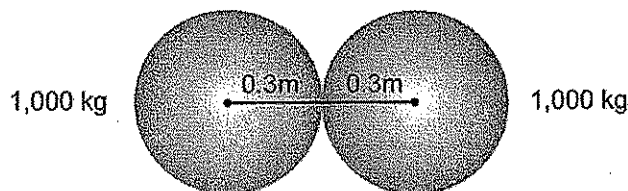
#### Law of universal gravitation

$$\text{Force (N)} \quad F = G \frac{m_1 m_2}{r^2}$$

Mass 1, Mass 2 (kg)  $m_1$   $m_2$   
 ↑ ↑ ↑  
 Gravitational Constant Distance between  
 ( $6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$ ) masses (m)



So, when the masses  $m_1$  and  $m_2$  are given in kilograms and the distance  $r$  is given in meters, the force has the unit of newtons. Remember that the distance  $r$  corresponds to the distance between the center of gravity of the two objects.



For example, the gravitational force between two spheres that are touching each other, each with a radius of 0.3 meter and a mass of 1,000 kilograms, is given by:

$$F = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2 \frac{1,000 \text{ kg} \times 1,000 \text{ kg}}{(0.3 \text{ m} + 0.3 \text{ m})^2} = 0.000185 \text{ N}$$

**Note:** A small car has a mass of approximately 1,000 kilograms. Try to visualize this much mass compressed into a sphere with a diameter of 0.3 meters (30 centimeters). If two such spheres were touching one another, the gravitational force between them would be only 0.000185 newtons. On Earth, this corresponds to the weight of a mass equal to only 18.9 milligrams. The gravitational force is not very strong!


**PRACTICE**

Answer the following problems. Write your answers using scientific notation.

1. Calculate the force between two objects that have masses of 70 kilograms and 2,000 kilograms separated by a distance of 1 meter.
2. Calculate the force between two touching grapefruits each with a radius of 0.08 meters and a mass of 0.45 kilograms.
3. Calculate the force between one grapefruit as described above and Earth. Earth has a mass of  $5.9742 \times 10^{24}$  kg and a radius of  $6.3710 \times 10^6$  meters. Assume the grapefruit is resting on Earth's surface.
4. A man on the moon with a mass of 90 kilograms weighs 146 newtons. The radius of the moon is  $1.74 \times 10^6$  meters. Find the mass of the moon.
5. For  $m = 5.9742 \times 10^{24}$  kilograms and  $r = 6.378 \times 10^6$  meters, what is the value given by this equation:  $G \frac{m}{r^2}$ ?
  - a. Write down your answer and simplify the units.
  - b. What does this number remind you of?
  - c. What real-life values do  $m$  and  $r$  correspond to?
6. The distance between Earth and its moon is  $3.84 \times 10^8$  meters. Earth's mass is  $m = 5.9742 \times 10^{24}$  kilograms and the mass of the moon is  $7.36 \times 10^{22}$  kilograms. What is the force between Earth and the moon?
7. A satellite is orbiting Earth at a distance of 35 kilometers. The satellite has a mass of 500 kilograms. What is the force between the planet and the satellite?
8. The mass of the sun is  $1.99 \times 10^{30}$  kilograms and its distance from Earth is 150 million kilometers ( $150 \times 10^9$  meters). What is the gravitational force between the sun and Earth?

$$\textcircled{1} F_g = \frac{(6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2})(70 \text{ kg})(2000 \text{ kg})}{1^2} = \boxed{9.338 \times 10^{-6} \text{ N}}$$

$$\textcircled{2} F_g = \frac{(6.67 \times 10^{-11})(0.45 \text{ kg})(0.45)}{(0.16)^2} = \boxed{5.27 \times 10^{-10} \text{ N}}$$

$$\textcircled{3} F_g = \frac{6.67 \times 10^{-11} (0.45)(5.9742 \times 10^{24} \text{ kg})}{(6.3710 \times 10^6 + 0.08)^2} = \boxed{4.418 \text{ N}}$$

#4 see ~~below~~

or

$$(0.45) 9.8 \frac{\text{m}}{\text{s}^2} = \boxed{4.41 \text{ N}}$$

$$\textcircled{5} \textcircled{a} \frac{6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \times 5.9742 \times 10^{24} \text{ kg}}{(6.378 \times 10^6 \text{ m})^2} = \boxed{9.796 \frac{\text{m}}{\text{s}^2}}$$

(b) Accel. due to gravity on earth

(c) Mass of earth  
Radius of earth

$$\textcircled{6} F_g = \frac{(6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2})(5.9742 \times 10^{24} \text{ kg})(7.36 \times 10^{22} \text{ kg})}{(3.84 \times 10^8 \text{ m})^2} = \boxed{1.99 \times 10^{20} \text{ N}}$$

$$\textcircled{4} M = 90 \text{ kg}$$

$$F_g = 146 \text{ N}$$

$$M_{\text{moon}} = ?$$

$$r = (1.74 \times 10^6 \text{ m})$$

$$F_g = \frac{G m M_{\text{moon}}}{r^2}$$

$$146 \text{ N} = \frac{(6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2})(90 \text{ kg}) M_{\text{moon}}}{(1.74 \times 10^6 \text{ m})^2}$$

$$\frac{(146)(1.74 \times 10^6)^2}{(6.67 \times 10^{-11})(90)} = \boxed{M_{\text{moon}} = 7.36 \times 10^{22} \text{ kg}}$$

$$\textcircled{7} \quad d = 35,000 \text{ m}$$

$$r_{\text{earth}} = 6.378 \times 10^6 \text{ m (from \#5)} \quad \left. \vphantom{r_{\text{earth}}} \right\} + \rightarrow r$$

$$M_s = 500 \text{ kg}$$

$$M_e = 5.9742 \times 10^{24} \text{ kg}$$

$$F_g = \frac{(6.67 \times 10^{-11})(500)(5.9742 \times 10^{24})}{(35000 + 6.378 \times 10^6)^2} = \boxed{4892.5 \text{ N}}$$

$\textcircled{8}$

$$F_g = \frac{(6.67 \times 10^{-11})(1.99 \times 10^{30})(5.9742 \times 10^{24})}{(150 \times 10^9)^2} = \boxed{3.524 \times 10^{22} \text{ N}}$$